

Janardan Bhagat Shikshan Prasarak Sanstha's
CHANGU KANA THAKUR
ARTS, COMMERCE \& SCIENCE COLLEGE, NEW PANVEL (AUTONOMOUS)

Re-accredited 'A+' Grade by NAAC
'College with Potential for Excellence' Status Awarded by UGC
'Best College Award' by University of Mumbai

Program: B.Sc.
Revised Syllabus of S.Y.B.Sc. Mathematics
Choice Based Credit \& Grading System (75:25)
w.e.f. Academic Year 2020-21

JANARDAN BHAGAT SHIKSHAN PRASARAK SANSTHA'S

## CHANGU KANA THAKUR

ART'S, COMMERCE AND SCIENCE COLLEGE, NEW PANVEL AUTONOMOUS

BOARD OF STUDIES IN MATHEMATICS MATHEMATICS FROM THE ACADEMIC YEAR 2020-2021


## S.Y.B.Sc.

## Introduction:

Mathematics pervades all aspects of life, whether at home, in civic life or in the workplace. It has been central to nearly all major scientific and technological advances. Many of the developments and decisions made in our community rely to an extent on the use of mathematics. Besides foundation skills and knowledge in mathematics for all citizen in the society, it is important to widen mathematical experience for those who are mathematically inclined.

## Aims and Objectives:

1. Giving students sufficient knowledge of fundamental principles, methods and a clear perception of boundless power of mathematical ideas and tools and know how to use them by analysing, modeling, solving and interpreting.
2. Reflecting on the broad nature of the subject and developing mathematical tools for continuing further study in various fields of science
3. Enhancing students overall development and to equip them with mathematical modeling abilities, problem solving skills, creative talent and power of communication necessary for various kinds of employment
4. A student should get adequate expossure to global and local concerns by looking at many aspects of mathematical Sciences

## Outcomes:

1. Students knowledge and skills will get enhanced and they will get confidence and interest in mathematics, so that they can master mathematics effectively and will be able to formulate and solve problems from mathematical perspective.
2. Students thinking ability and attitude will change towards learning mathematics and practicals will improve their logical and analytical thinking.


## Teaching Pattern for Semester-III

1. Three lectures per week per course. Each lecture is of 48 minutes duration.
2. One Practical (2L) per week per batch for courses USC3MT1, USC3MT2 combined and one Practical (3L) per week for course USC3MT3 (the batches to be formed as prescribed by the University). Each practical session is of 48 minutes duration.

## Teaching Pattern for Semester-IV

1. Three lectures per week per course. Each lecture is of 48 minutes duration.
2. One Practical (2L) per week per batch for courses USC3MT1, USC3MT2 combined and one Practical (3L) per week for course USC3MT3 (the batches to be formed as prescribed by the University). Each practical session is of 48 minutes duration.


## Scheme of Examination

Faculty of Science

## (Undergraduate Programmes)

## Credit Based Evaluation System

The performance of the learners shall be evaluated into two components. The learners Performance shall be assessed by Internal Assessment with $25 \%$ marks in the first component by conducting the Semester End Examinations with $75 \%$ marks in the second component. The allocation of marks for the Internal Assessment and Semester End Examinations are as shown below:-

## (A) Internal Assessment: 25\% (25 Marks)

| Sr. No. | Particular | Marks |
| :--- | :--- | :--- |
| 01 | One periodical class test / online examination to be conducted in <br> the given semester | 20 Marks |
| 02 | Active participation in routine class instructional deliveries and <br> overall conduct as a responsible learner, mannerism and articulation <br> and exhibit of leadership qualities in organizing related academic <br> activities | 05 Marks |

## Question Paper Pattern

(Periodical Class Test for the Courses at Under Graduate Programmes)
Maximum Marks: 20
Questions to be set: 02
Duration: 40 Minutes
All Questions are Compulsory

| Sr. No. | Particular | Marks |
| :--- | :--- | :--- |
| Q-01 | Match the Column / Fill in the Blanks / Multiple Choice Ques- <br> tions/ Answer in One or Two Lines (Concept based Questions) (1 <br> Marks / 2 Marks each) | 10 Marks |
| Q-02 | Answer in Brief (Attempt any Two of the Three) (5 Marks each) | 10 Marks |

## (B) Semester End Examination: 75\% (75 Marks)

Duration: The examination shall be of $2 \frac{1}{2}$ hours duration.

## Question Paper Pattern

| Sr. No. | Particular |
| :--- | :--- |
| 1 | There shall be four questions. |
| 2 | On each unit there will be one question and fourth question will be based on <br> entire syllabus. |
| 3 | Question number 1,2 and 3 will be of 20 marks each ( 40 marks with internal <br> options) and question number 4 will be of 15 marks ( 30 marks with internal <br> options). |
| 4 | All questions shall be compulsory with internal options. <br> 5Question may be subdivided into sub-questions $a, b, c, \cdots$ and the allocation <br> of marks depends on the weightage of the unit. |

## Passing Standard

The learners to pass a course shall have to obtain a minimum of $40 \%$ marks in aggregate for each course where the course consists of Internal Assessment and Semester End Examination. The learners shall obtain minimum of $40 \%$ marks (i.e. 10 out of 25 ) in the Internal Assessment and $40 \%$ marks in Semester End Examination (i.e. 30 Out of 75) separately, to pass the course and minimum of Grade D, wherever applicable, to pass a particular semester. A learner will be said to have passed the course if the learner passes the Internal Assessment and Semester End Examination together.

## Semester End Practical Examinations

At the end of the Semesters III \& IV, Practical examinations of three hours duration and 150 marks shall be conducted for the courses USC3MTP, USC4MTP.

In semester III, the Practical examinations for USC3MT1 and USC3MT2 are held together by the college. The Practical examination for USC3MT3 is held separately by the college.

In semester IV, the Practical examinations for USC4MT1 and USC4MT2 are held together by the college. The Practical examination for USC4MT3 is held separately by the college.

## Paper Pattern

The question paper shall have three parts $\mathrm{A}, \mathrm{B}, \mathrm{C}$.
Each part shall have two Sections.
Section I: Objective in nature: Attempt any Eight out of Twelve multiple choice questions. ( $8 \times 3=24$ Marks)
Section II: Problems: Attempt any Two out of Three.
$(8 \times 2=16$ Marks $)$

| Practical <br> Course | Part A | Part B | Part C | Marks out <br> of | Duration |
| :--- | :--- | :--- | :--- | :--- | :--- |
| USC3MTP | Questions from <br> USC3MT1 | Questions from <br> USC3MT2 | Questions from <br> USC3MT3 | 120 | 3 |
| USC4MTP | Questions from <br> USC4MT15 | Questions from <br> USC4MT2 | Questions from <br> USC4MT3 | 120 | 3 |

## Marks for Journals and Viva:

For each course USC3MT1, USC3MT2,USC3MT3 and USC4MT1, USC4MT2,USC4MT3

1. Journals: 05 marks.
2. Viva:05 marks.


## List of Courses for Semester-III

## PAPER I: CALCULUS-III

| Cource Code | Unit | Topic | Credit | Lecture <br> per Week |
| :--- | :--- | :--- | :--- | :--- |
| USC2MT1 | Unit I | Functions of several variables | 2 |  |
|  | Unit II | Differentiation |  |  |
|  | Unit III | Applications |  |  |

## PAPER II: ALGEBRA-III

| Cource Code | Unit | Topic | Credit | Lecture <br> per Week |
| :--- | :--- | :--- | :--- | :--- |
| USC2MT1 | Unit I | Vector spaces over $\mathbb{B}$. | 2 | 3 |
|  | Unit II | Linear Transformations and Ma- <br> trices |  |  |
|  | Unit III | Determinants |  |

## PAPER III: DISCRETE MATHEMATICS

| Cource Code | Unit <br> USC2MT1 | Unit I | Topic | Craphs |
| :--- | :--- | :--- | :--- | :--- |
|  | Unit II | Preliminary Counting | Lecture <br> per Week |  |
|  | Unit III | Advanced Counting | 2 | 3 |

PRACTICAL-III

| Cource Code | Part | Paper | Credit | Lecture <br> per Week |
| :--- | :--- | :--- | :--- | :--- |
| USC3MTP | A | USC3MT1 | 3 | 5 |
|  | B | USC3MT2 |  |  |
|  | C | USC3MT3 |  |  |



## List of Courses for Semester-IV

## PAPER I: CALCULUS-IV

| Cource Code | Unit | Topic | Credit | Lecture <br> per Week |
| :--- | :--- | :--- | :--- | :--- |
| USC2MT1 | Unit I | Riemann Integration |  | 3 |
|  | Unit II | Indefinite Integrals and Improper <br> Integrals |  | 3 |
|  | Unit III | Applications |  |  |

PAPER II: ALGEBRA-IV

| Cource Code | Unit | Topic | Credit | Lecture <br> per Week |
| :--- | :--- | :--- | :--- | :--- |
| USC2MT1 | Unit I | Inner Product Spaces | 2 | 3 |
|  | Unit II | Eigenvalues and Eigenvectors |  |  |
|  | Unit III | Diagonalization |  |  |

PAPER III: ORDINARY DIFFERENTIAL EQUATIONS

| Cource Code | Unit | Topic | Credit | Lecture <br> per Week |
| :--- | :--- | :--- | :--- | :--- |
| USC2MT1 Unit I | Second order differential equa- <br> tions |  |  |  |
|  | Unit II | Power Series solution of ordinary <br> differential equations | 2 | 3 |
|  | Unit III | Laplace Transform |  |  |

## PRACTICAL-IV

| Cource Code | Part | Paper | Credit | Lecture <br> per Week |
| :--- | :--- | :--- | :--- | :--- |
| USC4MTP | A | USC4MT1 | 3 | 5 |
|  | B | USC4MT2 |  |  |
|  | C | USC4MT3 |  |  |



## Syllabus for Semester-III



## USC3MT1: CALCULUS-III

## Note: All topics have to be covered with proof in details (unless mentioned otherwise) and with examples.

## 1. Unit I: Functions of several variables

(15 Lectures)
(a) The Euclidean inner product on $\mathbb{R}^{n}$ and Euclidean norm function on $\mathbb{R}^{n}$, distance between two points, open ball in $\mathbb{R}^{n}$, definition of an open subset of $\mathbb{R}^{n}$, neighborhood of a point in $\mathbb{R}^{n}$, sequences in $\mathbb{R}^{n}$, convergence of sequences- these concepts should be specically discussed for $\mathrm{n}=2$ and $\mathrm{n}=3$.
(b) Functions from $\mathbb{R}^{n} \rightarrow \mathbb{R}$ (scalar fields) and from $\mathbb{R}^{n} \rightarrow \mathbb{R}^{m}$ (vector fields), limits, continuity of functions, basic results on limits and continuity of sum, difference, scalar multiples of vector fields, continuity and components of a vector fields.
(c) Directional derivatives and partial derivatives of scalar fields.
(d) Mean value theorem for derivatives of scalar fields.

## 2. Unit II: Differentiation

(15 Lectures)
(a) Differentiability of a scalar field at a point of $\mathbb{R}^{n}$ (in terms of linear transformation) and on an open subset of $\mathbb{R}^{n}$, the total derivative, uniqueness of total derivative of a differentiable function at a point, simple examples of finding total derivative of functions such as $f(x, y)=x^{2}-y^{2}, f(x, y, z)=x+y+z$, differentiability at a point of a function $f$ implies continuity and existence of direction derivatives of $f$ at the point, the existence of contínuous partial derivatives in a neighborhood of a point implies differentiability at the point.
(b) Gradient of a scalar field, geometric properties of gradient, level sets and tangent planes.
(c) Chain rule for scalar fields.
(d) Second order partial derivatives, mixed partial derivatives, sucient condition for equality of mixed partial derivative.
3. Unit III: Applications
(a) Second order Taylors formula for scalar fields.
(b) Differentiability of vector fields, denition of differentiability of a vector field at a point, Jacobian matrix, differentiability of a vector field at a point implies continuity. The chain rule for derivative of vector fields (statements only)
(c) Mean value inequality.
(d) Hessian matrix, Maxima, minima and saddle points.
(e) Second derivative test for extrema of functions of two variables.
(f) Method of Lagrange Multipliers.

## Recommended Text Books:

1. T. Apostol: Calculus, Vol. 2, John Wiley.
2. J. Stewart, Calculus, Brooke/ Cole Publishing Co.

## Additional Reference Books

1. G.B. Thoman and R. L. Finney, Calculus and Analytic Geometry, Ninth Edition, AddisonWesley, 1998.
2. Sudhir R. Ghorpade and Balmohan V. Limaye, A Course in Multivariable Calculus and Analysis, Springer International Edition.
3. Howard Anton, Calculus- A new Horizon, Sixth Edition, John Wiley and Sons Inc, 1999.

## Suggested Practicals (Sem III)

1. Sequences in $\mathbb{R}^{2}$ and $\mathbb{R}^{3}$ limits and continuity of scalar fields and vector fields ,using definition and otherwise, iterated limits.
2. Computing directional derivatives, partial derivatives and mean value theorem of scalar fields.
3. Total derivative, gradient, level sets and tangent planes.
4. Chain rule, higher order derivatives and mixed partial derivatives of scalar fields.
5. Taylors formula, differentiation of a vector field at a point, nding Hessian/Jacobean matrix, Mean Value Inequality.
6. Finding maxima, minima and saddle points, second derivative test for extrema of functions of two variables and method of Lagrange multipliers.
7. Miscellaneous Theoretical Questions based on full paper

## USC3MT2: ALGEBRA-III

## Note: All topics have to be covered with proof in details (unless mentioned otherwise) and with examples.

1. Unit I: Vector Spaces over $\mathbb{R}$
(15 Lectures)
(a) Definition of a Vector Spaces over $\mathbb{R}$ and examples.
(b) Subspaces - definition and examples.
(c) The sum and intersection of subspaces, direct sum of vector spaces.
(d) Linear combination of vectors, convex sets, linear span of a subset of a vector space.
(e) Linear dependence and independence of a set.
(f) Basis of a vector space, basis as a maximal linearly independent set and a minimal set of generators. Dimension of a vector space.
2. Unit II: Linear Transforations and Matrices
(15 Lectures)
(a) Linear transformations: definition, properties and examples, Kernel and image of a linear transformation, Rank-Nullity theorem (with proof), Linear isomorphisms, inverse of a linear isomorphism, Matrix and linear transformation.
(b) The matrix units and elementary matrices.
(c) Row space, column space of an $m \times n$ matrix, row rank and column rank of a matrix.
(d) Equivalence of rank of an $m \times n$ matrix A and rank of the linear transformation $L_{A}: \mathbb{R}^{n} \rightarrow \mathbb{R}^{m}\left(L_{A}(A)=A X\right)$. The dimension of solution space of the system of linear equations $A X=0$ equals $n$ rank (A).
(e) The solutions of non-homogeneous systems of linear equations represented by $A X=$ $B$ and the general solution of the homogeneous system.
3. Unit III: Determinants
(a) Definition of determinant as an n-linear skew-symmetric function. Determinant of a matrix as determinant of its column vectors (or row vectors).
(b) Existence and uniqueness of determinant function via permutations.
(c) Laplace expansion of a determinant, Vandermonde determinant, determinant of upper triangular and lower triangular matrices.
(d) Linear dependence and independence of vectors in $\mathbb{R}^{\mathrm{m}}$ using determinants, The existence and uniqueness of the system $A X=B$, where A is an $n \times n$ matrix with $\operatorname{det}(A) \neq 0$.
(e) Cofactors and minors of a matrix, Adjoint of an $n \times n$ matrix A.
(f) Cramer's rule.
(g) Determinant as area and volume.

## Recommended Books:

1. Serge Lang: Introduction to Linear Algebra, Springer Verlag.
2. S. Kumaresan: Linear Algebra A geometric approach, Prentice Hall of India Private Limited.

## Additional Reference Books:

1. M. Artin: Algebra, Prentice Hall of India Private Limited.
2. K. Hoffman and R. Kunze: Linear Algebra, Tata McGraw-Hill, New Delhi.
3. Gilbert Strang: Linear Algebra and its applications, International Student Edition.
4. L. Smith: Linear Algebra, Springer Verlag.
5. A. Ramachandra Rao and P. Bhima Sankaran: Linear Algebra, Tata McGrawHill, New Delhi.
6. T. Banchoff and J. Wermer: Linear Algebra through Geometry, Springer Verlag Newyork, 198
7. Sheldon Axler : Linear Algebra done right, Springer Verlag, Newyork.
8. Klaus Janich : Linear Algebra.
9. Otto Bretcher: Linear Algebra with Applications, Pearson Education.
10. Gareth Williams: Linear Algebra with Applications, Narosa Publication.

## Suggested Practicals (Sem III)

1. Subspaces: Determine whether a given subset of a vector space is a subspace.
2. Linear dependence and independence of subsets of a vector space.
3. Rank-Nullity Theorem.
4. System of linear equations.
5. Determinant, calculating determinants of $2 \times 2$ matrices, $n \times n$ diagonal, upper triangular matrices using definition and laplace exapansion.
6. Finding inverses of Finding inverses of $n \times n$ matrices using adjoint
7. Determinant, calculating determinants of $2 \times 2$ matrices, $3 \times 3$ matrices using adjoint.
8. Miscellaneous Theoretical Questions based on full paper

## USC3MT3: DISCRETE MATHEMATICS

## Note: All topics have to be covered with proof in details (unless mentioned otherwise) and with examples.

## 1. Unit I: Graphs

(15 Lectures)
(a) Introduction to graphs: Types of graphs: Simple graph, directed graph, (One example/graph model of each type to be discussed).
(b) Graph Terminology: Adjacent vertices, degree of a vertex, isolated vertex, pendant vertex in a undirected graph, The handshaking Theorem for an undirected graph (statement only), Theorem: An undirected graph has an even number odd vertices (statement only).
(c) Some special simple graphs (by simple examples): Complete graph, cycle, wheel in a graph, Bipartite graph, regular graph.
(d) Representing graphs and graph isomorphism: Adjacency matrix of a simple graph, Incidence matrix of an undirected graph,
(e) Connectivity: Paths, circuits, simple paths, simple circuits in a graph (simple examples), Connecting paths between vertices (simple examples), Euler paths and circuits, Hamilton paths and circuits, Diracs Theorem (statement only), Ores Theorem (statement only), Planar graphs, planar representation of graphs, Eulers formula. Kuratowskis Theorem (statement only).
(f) Algorithms: Shortest path problem: Construction of Eulerian path by Fleurys Algorithm, The shortest path algorithm - Dijkstras Algorithm, Floyds Algorithm to find the length of the shortest path.
2. Unit II: Preliminary Counting
(15 Lectures)
(a) Finite and infinite sets, countable and uncountable sets with examples
(b) Addition and Multiplication Principle, counting sets of pairs, Two ways counting.
(c) Stirling numbers of second kind. Simple recursion formulae satisfied by $S(n ; k)$ for $k=1,2, \cdots, n 1, n$
(d) Pigeonhole principle and its strong form, its applications to geometry, monotonic sequences etc.

## 3. Unit III: Advanced Counting

(a) Binomial and Multinomial Theorem, Pascal identity, examples of standard identities with emphasis on combinatorial proofs.
(b) Permutation and combination of sets and multi-sets, circular permutations, emphasis on solving problems.
(c) Non-negative and positive solutions of equation $x_{1}+x_{2}+\cdots+x_{k}=n$
(d) Principal of inclusion and exclusion, its applications, derangements, explicit formula for $d_{n}$, deriving formula for Eulers function $\phi(n)$

## Recommended Books:

1. R. Wilson, Introduction to Graph theory, Fourth Edition, Prentice Hall.
2. K. H. Rosen, Discrete Mathematics and Its Applications, McGraw Hill Edition.
3. B. Kolman, Robert Busby, Sharon Ross: Discrete Mathematical Structures, Prentice-Hall India.
4. N. Biggs, Discrete Mathematics, Oxford.
5. Norman Biggs: Discrete Mathematics, Oxford University Press.
6. Richard Brualdi: Introductory Combinatorics, John Wiley and sons.
7. V. Krishnamurthy: Combinatorics-Theory and Applications, Affiliated East West Press.
8. Discrete Mathematics and its Applications, Tata McGraw Hills.
9. Schaums outline series: Discrete mathematics,
10. Applied Combinatorics: Allen Tucker, John Wiley and Sons

## Additional Reference Books:

1. D. B. West, Introduction to graph Theory, Pearson.
2. F. Harary, Graph Theory, Narosa Publication.
3. Graham, Knuth and Patashnik, Concrete Mathematics, Pearson Education Asia Low Price Edition.

## Suggested Practicals (Sem III)

1. Drawing a graph, counting the degree of vertices and number of edges.
2. Representing a given graph by an adjacency matrix and drawing a graph having given matrix as adjacency matrix.
3. Determining whether the given graph is connected or not. Finding connected components of a graph. Finding strongly connected components of a graph. Finding cut vertices.
4. Problems based on counting principles, Two way counting.
5. Stirling numbers of second kind, Pigeon hole principle.
6. Multinomial theorem, identities, permutation and combination of multi-set.
7. Inclusion-Exclusion principle. Euler phi function.
8. Miscellaneous theory quesitons from all units.


Syllabus for Semester-IV


## USC4MT1: CALCULUS-IV

Note: All topics have to be covered with proof in details (unless mentioned otherwise) and with examples.

## 1. Unit I: Riemann Integration

(15 Lectures)
Approximation of area, Upper/Lower Riemann sums and properties, Upper/Lower integrals, Definition of Riemann integral on a closed and bounded interval, Criterion of Riemann integrability, if $a<c<b$ then $f \in \mathbb{R}[a, b]$, if and only if $f \in \mathbb{R}[a, c]$ and $f \in R[c, b]$ and $\int_{a}^{b} f=\int_{a}^{c} f+\int_{c}^{b} f$ Properties:
(a) $f, g \in R[a, b] \Longrightarrow f+g, f-g, \lambda f \in R[a, b]$
(b) $\int_{a}^{b}(f+g)=\int_{a}^{b} f+\int_{a}^{b} g$
(c) $\int_{a}^{b} \lambda(f)=\lambda \int_{a}^{b} f$
(d) $f \in R[a, b] \Longrightarrow|f| \in R[a, b]$ and $\left|\int_{a}^{b} f\right| \leq \int_{a}^{b}|f|$
(e) If $f \geq 0$, andf $\in C[a, b] \Longrightarrow f \in R[a, b]$
(f) If f is bounded with finite number of discontinuities then $f \in R[a, b]$, generalize this if $f$ is monotone then $f \in R[a, b]$

## 2. Unit II : Indefinite and improper integrals

(15 lectures)
Continuity of $F(x)=\int_{a}^{x} f(t) d t$ where $f \in R[a, b]$, Fundamental theorem of calculus, Mean value theorem, Integrátion by parts, Leibnitz rule, Improper integrals type 1 and type 2, Absolute convergence of improper integrals, Comparison tests, Abels and Dirichlets tests (without proof).
3. Unit III : Applications
(a) $\beta$ and $\langle$ functions and their properties, relationship between $\beta$ and $\Gamma$ functions (without proof).
(b) Applications of definite Integrals : Area between curves, finding volumes by slicing, volumes of solids of revolution Disks and Washers, Cylindrical Shells, Lengths of plane curves, Areas of surfaces of revolution.

## References:

1. Calculus Thomas Finney, ninth edition section 5.1, 5.2, 5.3, 5.4, 5.5, 5.6.
2. R. R. Goldberg, Methods of Real Analysis, Oxford and IBH, 1964.
3. Ajit Kumar, S. Kumaresan, A Basic Course in Real Analysis, CRC Press, 2014.
4. T. Apostol, Calculus Vol.2, John Wiley.
5. K. Stewart, Calculus, Booke/Cole Publishing Co, 1994.
6. J. E. Marsden, A.J. Tromba and A. Weinstein, Basic multivariable calculus.
7. Bartle and Sherbet, Real analysis.

## Suggested Practicals (Sem IV)

1. Calculation of upper sum, lower sum and Riemann integral.
2. Problems on properties of Riemann integral.
3. Problems on fundamental theorem of calculus, mean value theorems, integration by parts, Leibnitz rule.
4. Convergence of improper integrals, applications of comparison tests, Abels and Dirichlets tests, and functions.
5. Beta Gamma Functions
6. Problems on area, volume, length.
7. Miscellaneous Theoretical Questions based on full paper

## USC4MT2: ALGEBRA-IV

## Note: All topics have to be covered with proof in details (unless mentioned otherwise) and with examples.

## 1. Unit I: Inner Product Spaces

(a) Dot product in $\mathbb{R}^{n}$, Definition of general inner product on a vector space over $\mathbb{R}$ and examples
(b) Norm of a vector in an inner product space. Cauchy-Schwarz inequality, Triangle inequality, Orthogonality of vectors, Pythagoras theorem and geometric applications in $\mathbb{R}^{2}$, Projections on a line, The projection being the closest approximation, Orthogonal complements of a subspace, Orthogonal complements in $\mathbb{R}^{2}$ and $\mathbb{R}^{3}$. Orthogonal sets and orthonormal sets in an inner product space, Orthogonal and orthonormal bases. Gram-Schmidt orthogonalization process, Simple examples in $\mathbb{R}^{3}, \mathbb{R}^{4}$
2. Unit II: Eigenvalues and eigenvectors
(15 Lectures)
(a) Eigenvalues and eigenvectors of a linear transformation $T: V \rightarrow V$, where V is a finite dimensional real vector space, Eigenvalues and eigenvectors of linear transformations examples.
(b) Eigenvalues of $n \times n$ real matrices.
(c) The linear independence of eigenvectors corresponding to distinct eigenvalues of a linear transformation.
(d) The characteristic polynomial of an $n \times n$ real matrix, characteristic roots.
(e) Similar matrices, characteristic polynomials of similar matrices.
(f) The characteristic polynomial of a linear transformation $T: V \rightarrow V$, where V is a finite dimensional real vector space.

## 3. UnitIII: Diagonalization

(a) Diagonalizability of an $n \times n$ real matrix and a linear transformation of a finite dimensional real vector space to itself. Definition : Geometric multiplicity and Algebraic multiplicity of eigenvalues of an $n \times n$ real matrix and of a linear transformation.
(b) An $n \times n$ matrix A is diagonalisable if and only if $\mathbb{R}^{n}$ has a basis of eigenvectors of A if and only if the sum of dimension of eigenspaces of A is n if and only if the algebraic and geometric multiplicities of eigenvalues of A coincide. Examples of non diagonalizable matrices.
(c) orthogonal diagonalization and Quadratic Forms.
(d) orthogonal diagonalization of $n \times n$ real symmetric matrices.

## Recommended Books:

1. Serge Lang: Introduction to Linear Algebra, Springer Verlag.
2. S. Kumaresan: Linear Algebra A geometric approach, Prentice Hall of India Private Limited.

## Additional Reference Books:

1. M. Artin: Algebra, Prentice Hall of India Private Limited.
2. K. Hoffman and R. Kunze: Linear Algebra, Tata McGraw-Hill, New Delhi.
3. Gilbert Strang: Linear Algebra and its applications, International Student Edition.
4. L. Smith: Linear Algebra, Springer Verlag.
5. A. Ramachandra Rao and P. Bhima Sankaran: Linear Algebra, Tata McGrawHill, New Delhi.
6. T. Banchoff and J. Wermer: Linear Algebra through Geometry, Springer Verlag Newyork, 198
7. Sheldon Axler : Linear Algebra done right, Springer Verlag, Newyork.
8. Klaus Janich : Linear Algebra.
9. Otto Bretcher: Linear Algebra with Applications, Pearson Education.
10. Gareth Williams: Linear Algebra with Applications, Narosa Publication.

## Suggested Practicals (Sem IV)

1. Inner Product Spaces, examples. Orthogonal complements in $\mathbb{R}^{2}$ and $\mathbb{R}^{3}$
2. Gram-Schmidt method
3. Finding characteristic polynomial, eigenvalues of $2 \times 2$ and $3 \times 3$ matrices.
4. Eigenvalues and eigenvectors of linear transformation
5. Diagonalization and orthogonal diagonalization.
6. Orthogonal Diagonalization Forms
7. Miscellaneous Theoretical Questions based on full paper

# USC4MT3: ORDINARY DIFFERENTIAL EQUATIONS 

## Note: All topics have to be covered with proof in details (unless mentioned otherwise) and with examples.

## 1. Unit I: Second order Linear Differential equations

(15 Leectures)
(a) First order and first degree differential equations
(b) Homogeneous and non-homogeneous second order linear differentiable equations: The space of solutions of the homogeneous equation as a vector space. Wronskian and linear independence of the solutions. The general solution of homogeneous differential equations. The general solution of a non-homogeneous second order equation. Complementary functions and particular integrals.
(c) The homogeneous equation with constant coefficients. auxiliary equation. The general solution corresponding to real and distinct roots, real and equal roots and complex roots of the auxiliary equation.
(d) Non-homogeneous equations: The method of undetermined coefficients. The method of variation of parameters.
2. Unit II: Power Series solution of ordinary differential equations (15 Lectures)
(a) A review of power series.
(b) Power series solutions of first order ordinary differential equations.
(c) Regular singular points of second order ordinary differential equations.
(d) Frobenius series solution of second order ordinary differential equations with regular singular points.
3. Unit HI: Laplace Transforms
(15 Lectures)
(a) Introduction, Properties of Laplace transform
(b) Laplace transform of elementary functions Problems using properties-Laplace transform of special function, unit step function and Dirac delta function
(c) Laplace transform of derivatives and Integrals, Evaluation of integral using Laplace Transform, Initial Value Theorem, Final Value Theorem and problems, Laplace Transform of periodic function
(d) Introduction, Properties of inverse Laplace transform, Problems (usual types)
(e) Convolution Theorem, Inverse Laplace Transform using Convolution theorem

## Recommended Books:

1. G. F. Simmons, Differential Equations with Applications and Historical Notes, McGraw Hill, 1972.
2. E. A. Coddington, An Introduction to Ordinary Differential Equations. Prentice Hall, 1961.
3. W. E. Boyce, R. C. DiPrima, Elementary Differential Equations and Boundary Value Problems, Wiely, 2013.
4. D. A. Murray, Introductory Course in Differential Equations, Longmans, Green and Co., 1897.
5. A. R. Forsyth, A Treatise on Differential Equations, MacMillan and Co., 1956.
6. Dr. S. Sreenath, S.Ranganatham, Dr. M.V.S.S.N.Prasad and Dr. V. Ramesh Babu, Fourier Series and Integral Transforms, S.Chand and Company Ltd,

## Additional Reference Books:

1. M.K. Venkataraman, Engineering Mathematics volume 3,National Publishing Co.
2. P.Kandasamy and others, Engineering Mathematics volume 3,S.Chand and Co.
3. Stanley Grossman and William R.Devit, $\downarrow$ Advanced Engineering Mathematics, Harper and Row publishers
4. Murray R Spiegel, Schaum's Outline of Laplace Transforms

## Suggested Practicals (Sem IV)

1. Finding general solution of homogeneous and non-homogeneous equations, use of known solutions to find the general solution of homogeneous equations.
2. Solving equations using method of undetermined coefficients and method of variation of parameters.
3. Power series solutions of first order ordinary differential equations.
4. Frobenius series method for second order ordinary differential equations.
5. Laplace transform of elementary functions
6. Laplace transform of derivatives and Integrals
7. inverse Laplace transform \& Convolution theorem
8. Miscellaneous

