



**Janardan Bhagat Shikshan Prasarak Sanstha's  
CHANGU KANA THAKUR  
ARTS, COMMERCE & SCIENCE COLLEGE,  
NEW PANVEL (AUTONOMOUS)**

**Re-accredited 'A+' Grade by NAAC  
'College with Potential for Excellence' Status Awarded by UGC  
'Best College Award' by University of Mumbai**

**Program: B.Sc**

**Revised Syllabus of F.Y.B.Sc. Mathematics  
Choice Based Credit & Grading System (75:25)  
w.e.f. Academic Year 2019-20**

<b>Sr. No.</b>	<b>Heading</b>	<b>Particulars</b>
1	Title of Course	Mathematics
2	Eligibility for Admission	12 <sup>th</sup> Science of all recognised Board
3	Passing marks	40%
4	Ordinances/Regulations (if any)	
5	No. of Semesters	Two
6	Level	U.G.
7	Pattern	Semester (75:25)
8	Status	Revised
9	To be implemented from Academic year	2019-2020

# F.Y.B.Sc.

## Introduction:

Mathematics pervades all aspects of life, whether at home, in civic life or in the workplace. It has been central to nearly all major scientific and technological advances. Many of the developments and decisions made in our community rely to an extent on the use of mathematics. Besides foundation skills and knowledge in mathematics for all citizen in the society, it is important to widen mathematical experience for those who are mathematically inclined.

## Aims and Objectives:

1. Giving students sufficient knowledge of fundamental principles, methods and a clear perception of boundless power of mathematical ideas and tools and know how to use them by analysing, modeling, solving and interpreting.
2. Reflecting on the broad nature of the subject and developing mathematical tools for continuing further study in various fields of science
3. Enhancing students overall development and to equip them with mathematical modeling abilities, problem solving skills, creative talent and power of communication necessary for various kinds of employment
4. A student should get adequate exposure to global and local concerns by looking at many aspects of mathematical Sciences

## Outcomes:

1. Students Knowledge and skills will get enhanced and they will get confidence and interest in mathematics, so that they can master mathematics effectively and will be able to formulate and solve problems from mathematical perspective.
2. Students thinking ability and attitude will change towards learning mathematics and practicals will improve their logical and analytical thinking.

## **Teaching Pattern for Semester-I**

1. Three lectures per week per course. Each lecture is of 48 minutes duration.
2. One Practical (2L) per week per batch for practical USMTP01 (the batches to be formed as pre scribed by the University).

## **Teaching Pattern for Semester-II**

1. Three lectures per week per course. Each lecture is of 48 minutes duration.
2. One Practical (2L) per week per batch for practical USMTP02 (the batches to be formed as pre scribed by the University).

## Scheme of Examination Faculty of Science (Undergraduate Programmes)

### Credit Based Evaluation System

The performance of the learners shall be evaluated into two components. The learners Performance shall be assessed by Internal Assessment with 25% marks in the first component by conducting the Semester End Examinations with 75% marks in the second component. The allocation of marks for the Internal Assessment and Semester End Examinations are as shown below:-

#### (A) Internal Assessment: 25% (25 Marks)

Sr. No.	Particular	Marks
01	One periodical class test / online examination to be conducted in the given semester	20 Marks
02	Active participation in routine class instructional deliveries and overall conduct as a responsible learner, mannerism and articulation and exhibit of leadership qualities in organizing related academic activities	05 Marks

### Question Paper Pattern

#### (Periodical Class Test for the Courses at Under Graduate Programmes)

Maximum Marks: 20

Questions to be set: 02

Duration: 40 Minutes

All Questions are Compulsory

Sr. No.	Particular	Marks
Q-01	Match the Column / Fill in the Blanks / Multiple Choice Questions/ Answer in One or Two Lines (Concept based Questions) ( 1 Marks / 2 Marks each)	10 Marks
Q-02	Answer in Brief (Attempt any Two of the Three) (5 Marks each)	10 Marks

## **(B) Semester End Examination: 75% (75 Marks)**

Duration: The examination shall be of  $2\frac{1}{2}$  hours duration.

### **Question Paper Pattern**

Sr. No.	Particular
1	There shall be four questions.
2	On each unit there will be one question and fourth question will be based on entire syllabus.
3	Question number 1, 2 and 3 will be of 20 marks each (40 marks with internal options) and question number 4 will be of 15 marks (30 marks with internal options).
4	All questions shall be compulsory with internal options.
5	Question may be subdivided into sub-questions $a, b, c, \dots$ and the allocation of marks depends on the weightage of the unit.

### **Passing Standard**

The learners to pass a course shall have to obtain a minimum of 40% marks in aggregate for each course where the course consists of Internal Assessment and Semester End Examination. The learners shall obtain minimum of 40% marks (i.e. 10 out of 25) in the Internal Assessment and 40% marks in Semester End Examination (i.e. 30 Out of 75) separately, to pass the course and minimum of Grade D, wherever applicable, to pass a particular semester. A learner will be said to have passed the course if the learner passes the Internal Assessment and Semester End Examination together.

## **Semester End Practical Examinations**

At the end of the Semesters I & II Practical examinations of two hours duration and 100 marks shall be conducted for the courses USC1MT1, USC1MT2

In semester I, the Practical examinations for USC1MT1 and USC1MT2 are held together.

In Semester II, the Practical examinations for USC2MT1 and USC2MT2 are held together.

### **Paper Pattern**

There will be Two Sections Section I & Section II for Mathematics Practical Question paper USC1MTP/USC2MTP

Section-I: Based on USC1MT1/USC2MT1

Section-II: Based on USC1MT2/USC2MT2

Maximum Marks: 80

Duration: 02 Hours

#### **Section I**

Q.1: Objective Questions- Attempt any 8 out of 12 multiple choice questions. ( $8 \times 3 = 24$ )

Q.2 : Problems- Attempt any two out of Three. ( $8 \times 2 = 16$ )

#### **Section II**

Q.1: Objective Questions- Attempt any 8 out of 12 multiple choice questions. ( $8 \times 3 = 24$ )

Q.2 : Problems- Attempt any two out of Three. ( $8 \times 2 = 16$ )

#### **Marks for Journals and Viva:**

For each course USC1MT1, USC1MT2 and USC2MT1, USC2MT2

1. Journals: 05 marks.
2. Viva: 05 marks.

## List of Courses for Semester-I

### CALCULUS-I

Course Code	Unit	Topic	Credit	Lecture per Week
USC1MT1	Unit I	Real Number System	2	3
	Unit II	Sequences		
	Unit III	Differential Equations		

### ALGEBRA-I

Course Code	Unit	Topic	Credit	Lecture per Week
USC1MT2	Unit I	Sets and functions	2	3
	Unit II	Integers and divisibility		
	Unit III	Theory of congruences		

### PRACTICAL-I

Course Code	Unit	Topic	Credit	Lecture per Week
USC1MTP	Section- I	USC1MT1	2	2
	Section II	USC1MT2		



## List of Courses for Semester-II

### CALCULUS-II

Course Code	Unit	Topic	Credit	Lecture per Week
USC2MT1	Unit I	Limits and Continuity	2	3
	Unit II	Continuous functions and Differentiation		
	Unit III	Applications of differentiation		

### ALGEBRA-II

Course Code	Unit	Topic	Credit	Lecture per Week
USC2MT2	Unit I	System of Linear Equations and Matrices	2	3
	Unit II	Permutations		
	Unit III	Polynomials		

### PRACTICAL-II

Course Code	Unit	Topic	Credit	Lecture per Week
USC2MTP	Section- I	USC2MT1	2	2
	Section II	USC2MT2		

# Syllabus for Semester-I

# USC1MT1: CALCULUS I

*Note: All topics have to be covered with proof in details (unless mentioned otherwise) and with examples.*

## Course Description:

We begin with a brief introduction of real numbers which will enable students to understand properties of real numbers. We will introduce real numbers and properties which will help students to understand the origin of number system. Basic theorems of real analysis like Archimedean property, Hausdorff property with applications will be introduced. After this we start with sequence of real numbers and concept of convergent sequences that will help students understand and solve problems which are widely prevalent in all branches of science. We introduced differential equation in next unit which has variety of applications in real world.

## Unit I : Real Number System (15 Lectures)

1. Real number system  $\mathbb{R}$  and order properties of  $\mathbb{R}$ , absolute value  $||$  and its properties.
2. AM-GM inequality, Cauchy-Schwarz inequality, Intervals and neighbourhoods, Hausdorff property.
3. Bounded sets, statements of l.u.b. axiom and its consequences, Supremum and Infimum, Maximum and Minimum, Archimedean property and its applications, density of rationals.

## Unit II : Sequences (15 Lectures)

1. Definition of a sequence and examples, Convergence of sequences, every convergent sequences is bounded. Limit of a convergent sequence and uniqueness of limit, Divergent sequences.
2. Convergence of standard sequences like (i)  $\frac{1}{1+na}$  where  $a > 0$ , (ii)  $b^n$  where  $0 < b \leq 1$ , (iii)  $c^{\frac{1}{n}}$  where  $c > 0$ , (iv)  $n^{\frac{1}{n}}$
3. Algebra of convergent sequences, sandwich theorem, monotone sequences, monotone convergence theorem and consequences as convergence of  $(1 + \frac{1}{n})^n$
4. Definition of subsequence, subsequence of a convergent sequence is convergent and converges to the same limit, definition of a Cauchy sequences, every convergent sequences s a Cauchy sequence and converse

## Unit III :Differential Equations (15 Lectures)

1. Solutions of homogeneous and non-homogeneous differential equations of first order and first degree, Notion of partial derivative, solving exact differential equations.
2. Rules for finding integrating factor (I.F) (without proof ) for non-exact equations such as:

- (a)  $\frac{1}{Mx+Ny}$  is an I.F if  $Mx + Ny \neq 0$  and  $Mdx + Ndy$  is homogeneous
  - (b)  $\frac{1}{Mx-Ny}$  is an I.F if  $Mx - Ny \neq 0$  and  $Mdx + Ndy$  is of the type  $f_1(xy)ydx + f_2(xy)xdy = 0$
  - (c)  $e^{\int f(x)dx}$  is an I.F if  $N \neq 0$  and  $\frac{1}{N}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right)$  is a function of  $x$  alone say  $f(x)$
  - (d)  $e^{\int g(y)dy}$  is an I.F if  $M \neq 0$  and  $\frac{1}{M}\left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right)$  is a function of  $y$  alone say  $g(y)$
3. Finding solutions of first order differential equations of the type  $\frac{dy}{dx} + P(x)y = Q(x)y^n$  for  $n \geq 0$ . Applications to orthogonal trajectories, population growth, and finding the current at a given time.

### Reference Books

1. K.G. Binmore, Mathematical Analysis, Cambridge University Press, 1982.
2. R.R. Goldberg, Methods of Real Analysis, Oxford and IBH, 1964.
3. G. F. Simmons, Differential equations with applications and historical notes, McGraw Hill.
4. E. A. Coddington, An introduction to ordinary differential equations, Dover Books.
5. R. G. Bartle- D. R. Sherbert, Introduction to Real Analysis, John Wiley & Sons, 1994.

### Additional Reference Books

1. T.M. Apostol, Calculus Volume I, Wiley & Sons (Asia) Pte, Ltd.
2. Richard Courant-Fritz John, A Introduction to Calculus and Analysis, Volume I, Springer.
3. Ajit kumar- S. Kumaresan, A Basic Course in Real Analysis, CRC Press, 2014.
4. James Stewart, Calculus, Third Edition, Brooks cole Publishing Company, 1994.
5. Ghorpade, Sudhir R.-Limaye, Balmohan V., A Course and Real Analysis, Springer International Ltd.2000.

# USC1MT2: ALGEBRA I

*Note: All topics have to be covered with proof in details (unless mentioned otherwise) and with examples.*

## Course Description:

The aim of this course is to introduce students to basic concepts like sets, relations, equivalence relations and functions, etc. It will make students learn different techniques of proving theorems, lemmas using induction, proof by contradiction etc. We also equip them with integers, division algorithm, congruences and its applications.

## Unit I : Sets and functions (15 Lectures)

1. Negation of a statement, use of quantifiers, sets, union and intersection of sets, complement of a set, De Morgans law, Cartesian product of sets.
2. Definition of a function; domain, co-domain and range of a function, composite functions, examples, Graph of a function, Injective, surjective, bijective functions; composite of injective, surjective, bijective functions when defined.
3. Invertible functions, bijective functions are invertible and conversely. Examples of functions including constant, identity, projection, inclusion.
4. Image and inverse image of a set under  $f$  interrelated with union, intersection and complement. Finite and infinite sets. Countable set and its examples such as  $\mathbb{Z}$ ,  $\mathbb{Q}$ . Uncountable set and its examples.

## Unit II : Integers and divisibility (15 Lectures)

1. Well-ordering property, First and second principle of mathematical induction as a consequence of well-ordering property
2. Divisibility in integers, division algorithm, existence & uniqueness of greatest common divisor (g.c.d.) and least common multiple (l.c.m.) and their basic properties. Bezouts identity and its applications.
3. Euclidean algorithm, Primes, Euclids lemma, Fundamental theorem of arithmetic, The set of primes is infinite.
4. The necessary and sufficient condition to have a solution for the linear Diophantine equation  $ax + by = c$ . Solving of linear Diophantine equation with examples.

## Unit III : Theory of congruences (15 Lectures)

1. Equivalence relation, equivalence classes and properties, Definition of a partition, every partition gives an equivalence relation and vice versa. Congruences, definition and elementary properties, Congruence is an equivalence relation on  $\mathbb{Z}$ , residue classes and partition of  $\mathbb{Z}$ , addition modulo  $n$ , multiplication modulo  $n$ , examples
2. Linear congruences. Chinese remainder theorem and its applications

3. Eulers  $\phi$  function, Eulers theorem, Fermats little theorem, Wilsons theorem and their applications.

### **Reference Books**

1. David M. Burton, Elementary Number Theory, Seventh Edition, McGraw Hill Education (India) Private Ltd.
2. S. Kumaresan, Ajit Kumar and Bhaba Kumar Sarma, A foundation course in Mathematics, 2018 edition, Narosa publication house.
3. David M. Burton, Elementary number theory, seventh edition, Tata McGraw-Hill edition.

### **Additional Reference Books**

1. Ivan Niven, Herbert S. Zuckerman, Introduction to the theory of numbers, fifth edition, Wiley eastern limited.
2. R.G. Bartle and D.R. Sherbert, Introduction to real analysis, third edition, John Wiley and Sons.
3. Jones and Jones, Elementary number theory, second edition, Springer
4. I.S. Luthar, Sets, functions and numbers, 2005 edition, Narosa publishing house.
5. Thomas Koshy, Elementary number theory with applications, Academic press

## USC1MTP: MATHEMATICS PRACTICAL-I

### (A) Practicals for USC1MT1

- (I) Application based examples of Archimedean property, intervals, neighbourhood. Consequences of l.u.b axiom, infimum and supremum of sets.
- (II) Problems on convergent sequences, subsequences and Cauchy sequences
- (III) Solving exact, non- exact and Bernoulli's differential equations.
- (IV) Miscellaneous Theoretical Questions based on full paper.

### (B) Practicals for USC1MT2

- (I) Problems on countability, Functions (image and inverse image), injective, surjective, bijective functions, finding inverses of bijective functions.
- (II) Problems on Mathematical induction, Division Algorithm and Euclidean algorithm in  $\mathbb{Z}$ , fundamental theorem of arithmetic and solving linear Diophantine equations.
- (III) Problems on congruences, equivalence relation and Chinese remainder theorem, Euler's function, Fermat's little theorem, Wilson's theorem.
- (IV) Miscellaneous Theoretical Questions based on full paper.

## Syllabus for Semester-II



## USC2MT1: CALCULUS II

*Note: All topics have to be covered with proof in details (unless mentioned otherwise) and with examples.*

**Course Description:** The aim of this course is to expose students to the beauty of limits, continuity and the concept of differentiation. The first unit is based on limits and continuity in which students learn the definition of continuity and sequential continuity and the equivalence between them. Problems based on these concepts are solved rigourously. The next unit is based on continuity and differentiability. Here students understand the notion of differentiation of a real valued function and mean value theorems. In the last section the emphasis is on applications of differentiability.

### Unit I : Limits and Continuity (15 Lectures)

Brief review: Domain and range of a function, injective function, surjective function, bijective function, composite of two functions, (when defined) Inverse of a bijective function

1. Graphs of some standard functions such as  $|x|, e^x, \log x, ax^2 + bx + c, \frac{1}{x}, x^n (n \geq 3), \sin x, \cos x, \tan x, \sin \frac{1}{x}, x^2 \sin \frac{1}{x}$  over suitable intervals of  $\mathbb{R}$
2. Definition of Limit of a function  $\lim_{x \rightarrow a} f(x)$ , evaluation of limit of simple functions using the  $\epsilon - \delta$  definition, uniqueness of limit if it exists, algebra of limits, limits of composite function, sandwich theorem, left-hand-limit  $\lim_{x \rightarrow a^-} f(x)$ , Right hand-limit  $\lim_{x \rightarrow a^+} f(x)$
3. Continuous functions: Continuity of a real valued function on a set in terms of limits, examples, Continuity of a real valued function at end points of domain, Sequential continuity, Algebra of continuous functions, discontinuous functions, examples of removable and essential discontinuity.

### Unit II : Continuous Functions and Differentiability (15 Lectures)

1. If  $f : [a, b] \rightarrow \mathbb{R}$  is continuous at  $x_0 \in [a, b]$  and  $f(x_0) > 0$  then there exists a neighbourhood  $N$  of  $x_0$  such that  $f(x) > 0 \forall x \in N$ , If  $f : [a, b] \rightarrow \mathbb{R}$  is continuous function then the image  $f([a, b])$  is a closed interval, Intermediate value theorem and its applications, Bolzano- Weierstrass theorem (statement only): A continuous function on a closed and bounded interval is bounded and attains its bounds.
2. Differentiation of real valued function of one variable: Definition of differentiation at a point of an open interval, examples of differentiable and non differentiable functions, differentiable functions are continuous but not conversely, chain rule , Higher order derivatives, Leibnitz rule, Derivative of inverse functions, Implicit differentiation (only examples)

### **Unit III : Applications of differentiation (15 Lectures)**

1. Definition of local maximum and local minimum, necessary condition, stationary points, second derivative test, examples, Graphing of functions using first and second derivatives, concave , convex , concave functions, points of inflection
2. Rolles theorem, Lagranges and Cauchys mean value theorems, applications and examples, Monotone increasing and decreasing function, examples,
3. L-Hospital rule without proof, examples of intermediate forms, Taylors theorem with Lagranges form of remainder with proof. Taylors polynomial and applications.

### **Reference Books**

1. R.R. Goldberg, Methods of Real Analysis, Oxford and IBH, 1964.
2. James Stewart, Calculus, Third Edition, Brooks Cole Publishing company, 1994.
3. T.M.Apostol, Calculus, Vol I, Wiley And Sons (Asia) Pte. Ltd.

### **Additional Reference Books**

1. Richard Courant- Fritz John, A Introduction to Calculus and Analysis, Volume-I, Springer.
2. Ajit Kumar- S.Kumaresan, A Basic course in Real Analysis, CRC Press, 2014.
3. Ghorpade, Sudhir R, -Limaye, Balmohan V, A course in Calculus and Real Analysis, Springer International Ltd, 2000.
4. K.G. Binmore, Mathematical Analysis, Cambridge University Press, 1982.
5. G.B.Thomas, Calculus, 12 th Edition 2009

## USC2MT2: ALGEBRA II

*Note: All topics have to be covered with proof in details (unless mentioned otherwise) and with examples.*

### Course Description:

The aim of this course is to introduce System of linear equations and matrices and to understand polynomials over  $\mathbb{R}$  and permutations of a set. The first unit is devoted to system of linear equations. The second unit is devoted to permutation. In this unit we introduce symmetries of triangle, rectangle and squares. We also introduced cycle and transposition in second unit. In the last unit we introduce polynomials over set of reals and complex numbers. We also introduce gcd of two polynomials over  $\mathbb{R}$ , Euclidean algorithm and solve problems based on that. Different techniques using rational root theorem will enable students to find roots of a polynomial.

### Unit I :System of Equations and Matrices (15 Lectures)

1. Parametric Equation of Lines and Planes , System of homogeneous and non homogeneous linear Equations, The solution of  $m$  homogeneous linear equations in  $n$  unknowns by elimination and their geometrical interpretation for  $(m, n) = (1, 2), (1, 3), (2, 2), (2, 2), (3, 3)$ ; Definition of  $n$ -tuple of real numbers, sum of  $n$ -tuples and scalar multiple of  $n$ -tuple. Deduce that the system of  $m$  homogeneous linear equations has a non trivial solution if  $m < n$  .
2. Matrices with real entries; addition, scalar multiplication of matrices and multiplication of matrices, transpose of a matrix, types of matrices: zero matrix, identity matrix, scalar matrix, diagonal matrix, upper and lower triangular matrices, symmetric matrix, skew symmetric matrix, invertible matrix; Identities such as  $(AB)^t = B^t A^t$  ,  $(AB)^{-1} = B^{-1} A^{-1}$
3. System of linear equations in matrix form , Elementary row operations , row echelon matrix, Gaussian elimination method, Gauss Seidal Method.

### Unit II : Permutations (15 Lectures)

1. Definition of a permutation of a set, Set of all permutations of the set  $\{1, 2, \dots, n\}$  i.e.  $S_n$  and its cardinality, Symmetries of an equilateral triangle, square, rectangle.
2. Cycles, Composition of permutations, properties of permutations such as every permutation of a finite set can be written as a cycle or a product of disjoint cycles, disjoint cycles commute.
3. Transpositions, Any permutation can be expressed as a product of transpositions, order of a permutation, sign of a permutation.

### Unit III : Polynomials (15 Lectures)

1. Definition of a polynomial, polynomials over the  $\mathbb{Q}, \mathbb{R}$  or  $\mathbb{C}$ , Algebra of polynomials, degree of polynomial, basic properties.

2. Division algorithm in  $\mathbb{R}[x]$  (without proof), and g.c.d of two polynomials and its basic properties (without proof), Euclidean algorithm (without proof), applications, Roots of a polynomial, relation between roots and coefficients, multiplicity of a root, Remainder theorem, Factor theorem
3. A polynomial of degree  $n$  has at most  $n$  roots, Complex roots of a polynomial in  $\mathbb{R}[x]$  occur in conjugate pairs, Statement of Fundamental Theorem of Algebra, A polynomial of degree  $n$  in  $\mathbb{C}[x]$  has exactly  $n$  complex roots counted with multiplicity, A non constant polynomial in  $\mathbb{R}[x]$  can be expressed as a product of linear and quadratic factors in  $\mathbb{R}[x]$ , necessary condition for a rational number  $\frac{p}{q}$  to be a root of a polynomial with integer coefficients, simple consequences such as  $\sqrt{p}$  is an irrational number where  $p$  is a prime number,  $n^{\text{th}}$  roots of unity, sum of all the  $n^{\text{th}}$  roots of unity.

### Reference Books

1. S. Kumaresan, Linear algebra, a geometric approach first edition, Prentice hall of India, 2009.
2. Joseph A. Gallian, Contemporary abstract algebra, fourth edition, Narosa publications.
3. John Fraleigh, A first course in abstract algebra, seventh edition, Pearson, 2013.

### Additional Reference Books

1. Norman L. Biggs, Discrete mathematics, second edition, Oxford university press.
2. I.N. Herstein, Topics in algebra, second edition, Wiley India edition.
3. Serge Lang, Introduction to linear algebra, second edition, Springer.

## USC2MTP: MATHEMATICS PRACTICAL-II

### (A) Practicals for USC2MT1

- (I) Calculating limits of sequences, Cauchy sequences, monotone sequences.
- (II) Limits of function and Sandwich theorem, continuous and discontinuous functions, Properties of continuous and differentiable functions. Higher order derivatives, Leibnitz theorem. Mean value theorems and its applications
- (III) Extreme values, increasing and decreasing functions. Applications of Taylors theorem and Taylors polynomials.
- (IV) Miscellaneous Theoretical Questions based on full paper

### (B) Practicals for USC2MT2

- (I) Solving homogeneous system of  $m$  equations in  $n$  unknowns by elimination for  $(m, n) = (1, 2), (1, 3), (2, 2), (2, 3), (3, 3)$ , row echelon form. Solving system  $Ax = B$  by Gauss elimination, Solutions of system of linear Equations.
- (II) permutations of a finite set, Symmetries, cycles, compositions of permutations, Permutation as a product of 2-cycles, order and sign of a permutation.
- (III) Problems on division algorithm and gcd of two polynomials, Problems based on factor theorem, remainder theorem and rational root theorem.
- (IV) Miscellaneous Theoretical Questions based on full paper